

A short history of memristor development

R. Stanley Williams
HP Labs

Historical Background

During the 1960's, Prof. Leon Chua, who was then at Purdue University, established the mathematical foundation for nonlinear circuit theory. This work was the basis for his classic 1969 textbook *Introduction to Nonlinear Network Theory* as well as a large number of papers he has published in refereed journals. He became very well known because of this work – in part, it led to his recruitment to the faculty of the Electrical Engineering and Computer Science department of UC Berkeley and a large number of awards, including the IEEE Gustav Robert Kirchhoff Award, nine other major awards from IEEE and other scholarly organizations, 9 honorary doctorates at major universities around the world, and numerous visiting professorships [<http://www.eecs.berkeley.edu/~chua/>]. In his IEEE biography, he is acknowledged as the father of nonlinear circuit theory and cellular neural networks (CNN) [http://www.ieeeahn.org/wiki/index.php/Leon_Chua].

As a result of his work on nonlinear circuit elements, Chua made an interesting observation. For traditional linear circuits, there are only three independent two-terminal passive circuit elements: the resistor R , the capacitor C and the inductor L . However, when he generalized the mathematical relations to be nonlinear, there was another independent differential relationship that in principle coupled the charge q that flowed through a circuit and the flux ϕ in the circuit, $d\phi = M dq$, that was mathematically different from the nonlinear resistance that coupled the voltage v to the current i , $dv = R di$. As a strictly mathematical exercise, he explored the properties of this potentially new nonlinear circuit element, and found that it was essentially a resistor with memory – it was a device that changed its resistance depending on the amount of charge that flowed through the device, and thus he called this hypothetical circuit element M a memristor. This conclusion was independent of any physical mechanism that might couple the flux and charge, and in fact he did not postulate any mechanism at all. Moreover, the memristor definition did not even require causality. In other words, the mathematical relationship between flux and charge could be the result of some other cause – any mechanism that led to the constraint embodied by the equation $d\phi = M dq$ would lead to a device with the properties of a memristor. He published these initial findings [L. O. Chua, Memristor – the missing circuit element, *IEEE Trans. Circuit Theory* **18**, 507-519 (1971)] essentially as a curiosity – it was not obvious at that time that such a circuit element existed. However, some people (as I did at first) have taken this paper too literally and thought that a memristor must involve a direct interaction of a charge with a magnetic flux – in fact, there was no such requirement or restriction in the memristor definition.

This issue was made much clearer in a second paper published with his then student Sung Mo Kang [L. O. Chua & S. M. Kang, Memristive devices and systems, *Proc. IEEE* **64**, 209-223 (1976)]. This study was a critical generalization of the simple memristor concept of the first paper, but it has not been cited with the frequency of the 1971 paper, so fewer people are aware of its implications. Chua and Kang introduced the fact that a 'memristive device' has a state variable (or variables), indicated by w , that describes the physical properties of the device at any time. A memristive system is characterized by two equations, the 'quasi-static' conduction equation that relates the voltage across the device to the current through it at any particular time, $v = R(w,i) i$, and the dynamical equation, which explicitly asserts that the state variable w is a time varying function f of itself and possibly the current through the device, $dw/dt = f(w,i)$. Neither the flux ϕ nor the charge q appears in either of these two equations, but it is relatively 'easy to show' that if both R and f are independent of the current i , the two equations reduce to the original definition of a memristor. Furthermore, the quasi-static conduction equation places a requirement on the current-voltage characteristic of the device – if a memristive system is driven with some type of cyclic

excitation, such as a sinusoidal current, the plot of the voltage vs. the current will be a Lissajous curve for which the voltage is always equal to zero when the current is zero, and vice versa. Chua called this curve a 'pinched hysteresis loop', and it has an important physical interpretation – neither a memristor nor a memristive system stores either charge or energy (like a capacitor, for example), but they do 'remember' their history because of their changing resistance. This 1976 paper showed many other properties of the generalized memristor and also discussed possible examples, but again this was a mathematical exercise that was independent of any physical mechanism at the time. The key result was that any electronic circuit element that displayed a pinched hysteresis loop in its current-voltage characteristic could be described mathematically by the two memristive system equations. This is actually very useful, because if one can identify the state variable with a physical property of a device and determine the functional dependence of the time derivative of the state variable on the state variable and either the voltage or current, then one has a complete description of the dynamical behavior of the device. Along with the quasi-static conduction equation, one then has a mathematical model that in principle can predict the properties of the device under any conditions. This is crucial to actually designing a circuit that would utilize such a device.

These publications were mainly dormant until our group re-introduced them to the research community via our paper [Strukov et al., The missing memristor found, *Nature* **453**, 80-83 (2008)]. As more researchers began to cite the original works, it became clear to most that the distinctions between 'memristors' and 'memristive systems' were unimportant, so Chua has recommended that the nomenclature be simplified by referring to both as memristors, since in fact the generalization was 'a trivial extension', but if a distinction between the two was needed to call the simpler device an 'original memristor' [L. Chua, Resistance switching memories are memristors, *Appl. Phys. A* **102**, 765-783 (2011)].

What is a Memristor?

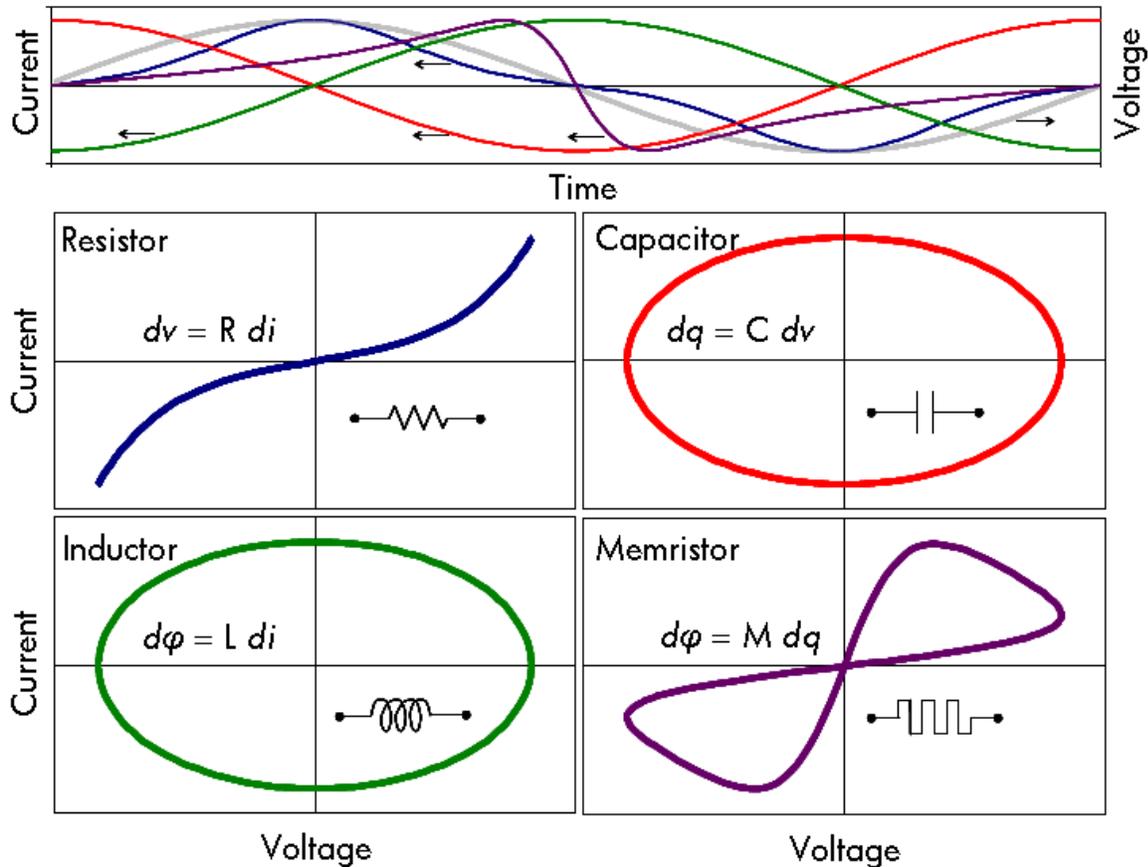
According to the general mathematical model, a memristor is any passive electronic circuit element that displays a pinched hysteresis loop in its i-v characteristic, independent of what the physical mechanism is that causes the hysteresis. The model is useful because it provides quantitative means to predict the properties of such a device in an electronic circuit, for example in a SPICE model. However, no mathematical model is perfect – it is only an approximation to real behavior – this is as true for a resistor as a memristor.

Examples of memristors include bipolar and unipolar resistive switches, often called RRAM or ReRAM; 'atomic switches'; spin-torque transfer RAM devices, phase-change memory devices, and several other systems based on a wide variety of materials and mechanisms [L. Chua, Resistance switching memories are memristors, *Appl. Phys. A* **102**, 765-783 (2011)]. For the most part, we have chosen to use the term memristor to describe the devices in our papers, not because we are trying to impose an 'HP brand' (especially since the term was invented by Leon Chua), but because we feel the general term connotes a broader range of applications. 'RAM' means random access memory, and that is certainly one application for memristors, but we find that much too restrictive, since they can also be used in a wide variety of other electronic circuits, including logic, FPGAs, and various types of 'synaptic' or 'neural' applications – memristors are much more than memory.

Why could a Memristor be considered a (Fourth) Fundamental Elemental Circuit Element?

Frankly, this is a judgment call. We will see how the textbooks choose to define it. However, there are some good arguments for why it should be considered the Fourth Fundamental Nonlinear Circuit Element. Chua has shown mathematically that it is not possible to construct an equivalent circuit for a memristor using any combination of only passive nonlinear resistors, capacitors and inductors. Thus, the memristor represents an independent 'basis function' for constructing passive nonlinear circuits, so it has a status similar to the nonlinear resistor, capacitor and inductor. The figure below is an illustration of this argument. The upper panel shows an applied voltage sine wave (gray) versus time with the corresponding

current for a resistor (blue), capacitor (red), inductor (green) and memristor (purple). The lower figures show the current-voltage characteristics for the four devices, with the characteristic pinched hysteresis loop of the memristor in the bottom right. It is nearly obvious by inspection that the memristor curve cannot be constructed by combining the others.



There are also arguments that there are far more than four fundamental electronic circuit elements. In fact, Chua has shown that there are essentially an infinite number of two-terminal circuit elements that can be defined via various integral and differential equations that relate voltage and current to each other [L. O. Chua, Nonlinear Circuit Foundations for Nanodevices, Part I: The Four-Element Torus. *Proc. IEEE* **91**, 1830-1859 (2003) – this is an interesting tutorial for the beginner], to which the memcapacitor and meminductor belong. It comes down to whether one wants to think of all of these possible circuit elements as being on an equal footing or choose the four lowest order relations to be a fundamental set with a large number of higher order cousins. Similar considerations apply in other fields – do we consider electrons, protons and neutrons fundamental or quarks or what?

Who 'Discovered' the Memristor?

The memristor as a mathematical model or entity was discovered and made rigorous by Leon Chua.

Independent of and even preceding his discovery, there were experimental observations of pinched hysteresis loops in two-terminal electrical measurements in a variety of material systems and subsequent development of devices based on those observations. We are not aware of any useful mathematical models presented in any of these previous works for predicting the behavior of these devices in an electronic circuit. We are not aware that any of these researchers cited Chua's papers after they appeared in print. In turn, Chua was not aware of these studies (except for one that he discussed in his 2003 paper cited above) – but this is not surprising, since he is an electronic circuit theorist and the experimental

studies appeared mainly in the materials science literature. There were no pointers across this disciplinary divide.

We never claimed to be the first to have observed these electrical characteristics. In our 2008 Nature paper, for example, we cited 20 papers by other researchers who had reported such observations and inventions, including the first example we knew (M. T. Hickmott in 1962) and a comprehensive review with over 150 references published in 1970 (G. Dearnaley, et al.). Several people have indicated to us that they were not included in our set of references, but in fact the number of references was limited by the publisher and any completely comprehensive list of references could number in the many hundreds if not thousands. In our papers, we cite those papers that appeared earliest and those that we have found most useful to our research. We have and continue to cite the work of others and our own work, as appropriate.

In summary, the memristor was a discovery – it is a rigorous mathematical model that can be used to predict the behavior of a wide variety of physical devices. There have been many developments of different types of memristors, now called by many different names, based on different materials and physical mechanisms, but they are all described by the same general mathematical formalism developed by Chua.

So, what did you guys at HP do, anyway?

Beginning in 1996, our group at HP Labs began a series of collaborations with Prof. James Heath, then at UCLA, to see if we could build nanoscale devices that we could use for alternatives to CMOS for logic and memory. A more complete history is presented in the following: [R. S. Williams, How we found the missing memristor, *IEEE Spectrum*, Dec. 8, 2008]. At that time, there were other groups in HP Labs that were researching MRAM and various oxide and polymer-based memories [e.g. S. Moller et al., A polymer/semiconductor write-once read-many-times memory, *Nature* **426**, 166-169 (2003)], so in fact HP has a large experience base and IP portfolio in this general area of memory devices. In order to distinguish ourselves from these other efforts, our group concentrated initially on molecular systems, with our first patent [US6128214] and publication [C. P. Collier et al., Electronically Configurable Molecular-Based Logic Gates, *Science* **285**, 391-394 (1999)] both submitted in 1999. We understood the need for a nonvolatile switch and concentrated on devices that yielded a pinched hysteresis loop, which we called 'bowties' at the time since we had not discovered Chua's papers yet. Over the next several years, we worked mainly on architectural and circuit concepts while trying to improve the molecular devices and exploring other material systems. We finally decided that molecules were not robust enough for applications and concentrated on inorganic systems. In the meantime, we began reading Chua's papers and trying to understand what was the cause of the pinched hysteresis loops in our devices. The big breakthrough, and our most significant contribution, came in 2006 when we realized that the time derivative of the state variable in Chua's dynamical state equation was comparable to the drift velocity of oxygen vacancies in a titanium dioxide resistive switch – we thus had a simple quantitative model based on a physical mechanism that we could use to explain the operation of the device and use in a predictive fashion in a circuit simulation. Although it had been right there in front of us for years, we had finally 'found' a memristor. Since then, we have been continually improving our memristor models, which has helped us understand the devices better, engineer them to improve their performance, and design circuits that take advantage of their properties.

So, in the end, magnetic flux had no apparent relation with the operation of titanium dioxide memristors. That does not mean that there are no devices in which a charge interacting with a flux would yield a memristor – I just don't know of an example. Any mechanism that is mathematically consistent with Chua's equations defines a memristor. That includes the drift-diffusion mechanism that we have postulated and are continually refining for titanium dioxide based devices [D. B. Strukov et al., Coupled Ionic and Electronic Transport Model of Thin-Film Semiconductor Memristive Behavior, *Small* **5**, 1058-63 (2009) and M. D. Pickett et al., Switching Dynamics in Titanium Dioxide Memristive Devices, *J. Appl. Phys.* **106**, 074508 (2009)]. We continue to explore this material system and several others as well, as do other

research groups. This is the nature of scientific research – we all learn from each other, taking insights from others and adding some of our own to hopefully advance the state of the art. We each try to distinguish what we are doing from what others are doing, to establish some type of priority. In the end, it is the peer review system that keeps the system in balance – any paper submitted will most likely be reviewed by your most critical competitor, so what finally appears in print has been vetted thoroughly. The community as a whole thus makes much more rapid progress than if we were each operating in a vacuum. If other researchers find our publications useful, they will use some of the concepts in their own work – if not, they won't. In a little over three years, our papers have been cited over 1000 times by other researchers in the field, so that is some indication that they are at least being read.